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Prisoners' dilemma in real-world acquaintance networks: Spikes and quasiequilibria induced by the interplay between structure and dynamics

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We study Nowak and May's spatial prisoners' dilemma game driven by mutations (random choices of suboptimal strategies) on empirical social networks. The time evolution of the cooperation level is highly complex containing spikes and steps between quasistable levels. A statistical characterization of the quasistable states and a study of the mechanisms behind the steps are given. We argue that the crucial structural ingredients causing the observed behavior is an inhomogeneous degree distribution and that the connections within vertices of highest degree are rather sparse. Based on these observations we construct model networks with a similar complex time evolution of the cooperation level.

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The prisoners' dilemma (PD) game is a powerful metaphor for the situation where mutual trust and cooperation is beneficial in a long perspective but egoism and guile can produce big short-term profit. One of the major achievements has been to establish the condition for cooperative behavior when two players meet repeatedly [1]. Another direction has been to investigate the criteria for cooperation to be stable in social space. In this approach the spatial PD game by Nowak and May [2] has been the basic model; a model where there are a number N of players interacting only with players in its immediate surrounding. Traditionally, spatial games have been studied on regular lattices; but real social networks are very complexly organized [3]—being random to some extent, but also having structure reflecting the social forces. As a step in this direction people have studied the spatial PD on more realistic model networks [4,5]. In this paper we pursue this idea to the end and let the players' encounters follow the ties of empirical social networks. The outcome is a very complex behavior arising from the interplay between the PD dynamics and the underlying network structure.

We represent the underlying networks as graphs $G=(V,E)$, where V is the set of N players (or vertices) and E is the set of M ties (or edges—unordered pairs of vertices) between them. The networks we use are mostly acquired from online interaction—through contacts within an Internet community and through email exchange. (A summary of the used networks can be seen in Table I.) Even if the structure of online interaction network differs from regular acquaintance networks, we believe that our results will hold for a quite large class of social networks. As a small test network we also use an acquaintance data constructed from a field survey (the "karate-club" network).

In Nowak and May's spatial PD game [2] each player, at each time step, adopts one of two strategies: cooperate C or defect D. To catch the dilemma one lets an encounter between two cooperators result in unity gain for both players, whereas a D-D encounter gives zero gain for both. However, if a cooperator meets a defector the cooperator scores zero and the defector scores $b\in(1,2)$ (b is called "temptation"). The gain $g(v)$ of a player v is the sum of the gains from the encounters with its neighbors. If a neighbor scores higher than a player, the player follows the high scoring neighbor. But, to drive the system (and model occasional irrational moves) a player chooses the opposite strategy with a probability p_m. We use synchronous updating of the players, i.e., one time step of the algorithm consists of one sweep (over all the players) to calculate the individual gains and one sweep to update the strategies.

Our key quantity is the cooperator density ρ—the fraction of players adopting the strategy C. In regular networks the cooperator level is characterized by random fluctuations and high-frequency oscillations (mostly of period two, so-called "blinkers" [2]). But, as seen in Fig. 1, when a regular underlying network is replaced by empirical social networks we get a complex behavior of both upward and downward spikes and steps between a number of quasiequilibrium levels. Other observations of quasistable states in games have been obtained for more complex update rules and larger strategy space [9–11] or for some special types of directed networks [5].

The steps between quasiequilibria in Fig. 1 suggest a scenario of relatively few stable states, where mutations on important vertices may cause a shift from one quasistable state to another. To get a better picture we plot a histogram over ρ (Fig. 2) that indeed has distinct peaks. For a given b the position and relative height of the peaks are p_m independent

TABLE I. Statistics of the networks: The number of vertices N, the number of edges M, the average degree in the maximal subgraph of the ten vertices of highest degree k_{10}, and the corresponding expectation value for random graphs of the same degree sequence $\overline{k_{10}}$.

Network	Reference	N	M	k_{10}	$\overline{k_{10}}$
pussokram.com	[6]	29 341	115 684	1.6	6.0(1)
Emails	[7]	40 346	58 224	1.8	4.9(1)
Karate club	[8]	34	78		

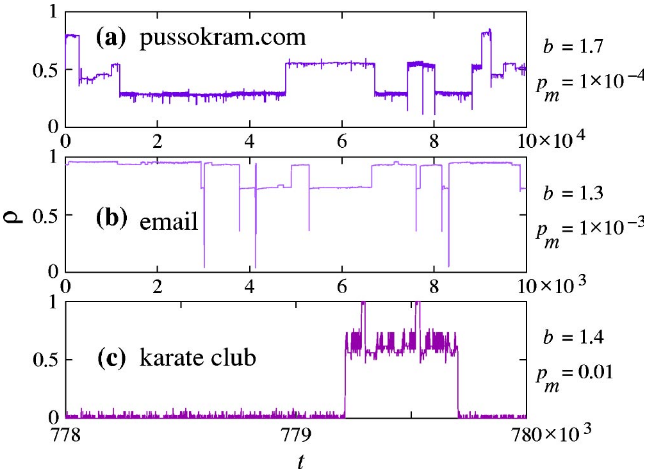

FIG. 1. Time evolution of the cooperator density ρ for different networks and values of the temptation b and mutation rate p_m parameters. All these time sequences are typical for the given set of parameter values except the karate-club network that spends most of the time in the all-D state $\rho \approx 0$.

(if p_m increases, the noise level of the histograms gets higher) and thus forms a kind of fingerprint of the network. We note that the histograms obtained from one long run or many shorter runs (with different random seeds) are the same, this indicates that the system is self-averaging.

Now we turn to the structure of the quasistable states. To start with a concrete example, in Fig. 3 we study the karate-club network at the intermediate ρ level of Fig. 1(d). In Fig. 3(a) [12] v_A and v_B scores highest among the border players (players with a neighbor of the opposite strategy). This makes many players, including the highly connected v_C, change strategy to C. In Fig. 3(b), the defector v_B has enough cooperators in their neighborhood to score higher than v_A, and thus complete the cycle. We note that a crucial point in sustaining the stability is that v_A and v_B are not connected. We also note that if v_D is mutated in Fig. 3(a) the system would loose its periodicity and stay in a constant configuration (until the next mutation). This suggests that the quasiequilibria in Fig. 1 does not correspond only to one state, but a set of states that are all close in Hamming distance.

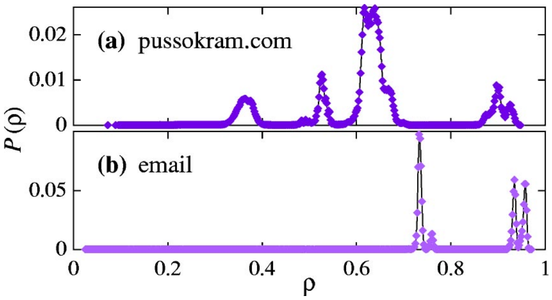

FIG. 2. Histograms (over 10^8 time steps) of the cooperator density. The temptation is 1.35 for both subfigures, the mutation rate is 0.001. Lines are guides is the eyes.

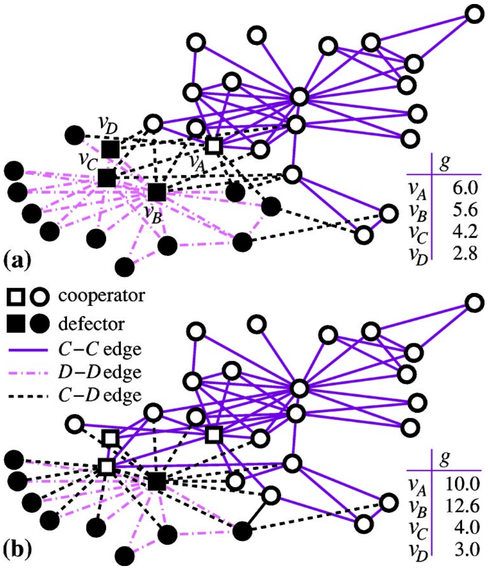

FIG. 3. The configurations of the intermediate quasistable state of the karate-club network from time steps 779 597 and 779 598 of the run in Fig. 1(d). The key players mentioned in the discussion are marked as squares.

How does the high-ρ states differ from the low-ρ states? From the update rule to follow the player with locally highest gain, one expects that defectors gain comparatively much in low ρ and vice versa. This is indeed true as shown in Fig. 4 where we plot the average defector gain g_D and cooperator gain g_C along with ρ. One may therefore talk about a high-ρ "cooperator controlled" and a low-ρ "defector controlled" state. The average defector gain is directly related to the fraction of boundary (C-D) edges ϕ—the total defector gain is the sum of all boundary edges times b, which gives the following expression for ϕ:

$$\phi = \frac{g_D(1-\rho)N}{bM}. \tag{1}$$

The values of g_D [seen in Fig. 4(c)] means that $\sim 40\%$ of the edges are border between cooperators and defectors in the defector controlled state. This rather many consider that the biggest cooperator cluster consists of $\sim 95\%$ of all cooperators (the same figure for defectors is $\sim 63\%$). These observations suggest numerous situations where many defec-

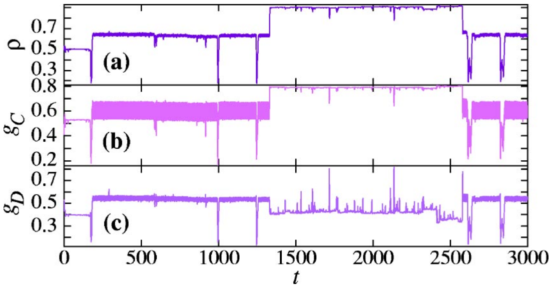

FIG. 4. (a) Cooperator density ρ. (b) Average cooperator gain g_C. (c) Average defector gain g_D. All curves are from the same run for the pussokram.com network and parameter values $p_m = 0.001$, $b = 1.35$.

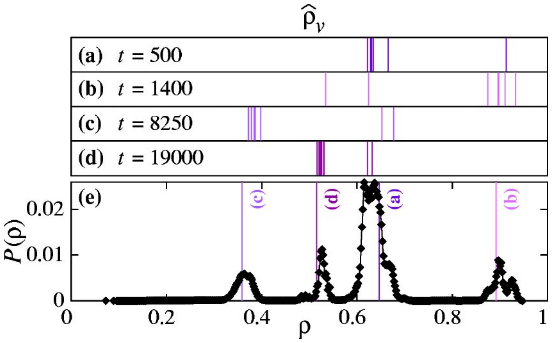

FIG. 5. The impact of one-player mutations. (a–d) shows ten $\hat{\rho}_v(t)$ (the eventual cooperator level if v, and no other vertex, is mutated and the system relaxed) that differs most from $\rho(t)$ at the indicated time steps. The original $\rho(t)$ is indicated together with the ρ histogram in (e).

tors exploit high-degree cooperators, such as v_C is being exploited in Fig. 3(b). (This picture is confirmed under general circumstances [13].)

To investigate the mechanism behind the steps of $\rho(t)$ we start by again considering the example of Fig. 3. If the most connected vertex v_B (with degree 17) is mutated from D to C in the configuration of Fig. 3(a) then $g(v_B)=4$ while $g(v_C)=5.6$. In the next time step v_B would shift to v_C's strategy D while v_C follows the cooperator v_A, so in fact all vertices except one degree-2 vertex will have the same strategies as without a mutation on v_B. If on the other hand, v_B was mutated in Fig. 3(b), all players would cooperate in the succeeding time step. By this example we conclude that a vertex' importance with respect to mutation is not only dependent on its degree but also on the current configuration.

For a little more quantitative approach to this problem we study the result of a one-player mutation as follows: Starting at time t we run the system without mutations until one period is completed—this is to start from a specific configuration c of the local attractor state [12]. Then we mutate vertex v and let the system evolve (without mutations) until one period is completed. Finally, we let the system yet complete a period and look for the configuration $\hat{c}$ with the minimal Hamming distance to c over the period (this is to make the difference zero if the system would relax back to the same state as prior to the mutation), and set $\hat{\rho}_v=\rho(\hat{c})$. In Figs. 5(a)–(d) we plot ten $\hat{\rho}_v$ that differs most from $\rho(c)$ at four different time steps in four different quasistable states. We observe that the system can change quasistable state due to mutations on single vertices, but at the same time not go to *any* other quasistable state. Furthermore, there is only a small number of mutations that actually causes a transition. Just as in the karate-club example above the configurations comprising a state may vary much from one time step to another. This fact explains the apparent lack of transitions to the $\rho \approx 0.35$ state—in fact, the transition to this state (around $t=8250$) occurs from a single-player mutation from a configuration in the $\rho \approx 0.65$ peak. As expected, all vertices contributing to the lines in Figs. 5(a)–(d) have high degree (the lowest degree is 111). But, as mentioned above, it is not true that a high degree implies a high importance—for example, of the ten most connected vertices, on average 5.5 contributes to the ten most different $\hat{\rho}_v$ values plotted in Figs. 5(a)–(d). It is thus not only the structure but the interplay between the structure and dynamics that gives rise to the quasistable equilibria. As a last point we note that the quasiequilibria are stable in the absence of mutations—if the system at the time steps of Figs. 5(a)–(d) is evolved without mutation, the largest shift in ρ is 0.008.

What features of the network structure are causing the nonstationary evolution of the cooperator density? As we have seen, the vertices have to have a highly skewed degree distribution—so that mutation on one vertex can cause many vertices, directly or indirectly, change strategy. Furthermore, it should not be the case that the vertices of highest degree all are interconnected—then these would all follow the same strategy and most of the other vertices too, so nothing more than small fluctuations of ρ, or steps involving almost all vertices, could occur. To test that there is a bias among the most connected vertices not to attach to each other, we sample different networks with the same degree sequence by randomly rewiring the original network with the restriction that the degree of each vertex is preserved [14,15]. The average degree of the maximal subgraph of the ten vertices of highest degree k_{10} and the expectation value of the same quantity for random graphs with the same degree sequence as G, $\overline{k_{10}}$, for the three large networks are shown in Table I. (The value ten is chosen arbitrarily—any number of the same order would do.) We see that k_{10} is indeed significantly lower than $\overline{k_{10}}$ for all three networks. We also note that if one runs the PD dynamics on a rewired network one sees spikes but no quasistable states [13].

Networks with a broad degree distribution can be generated by, for example, Barabási and Albert's (BA) scale-free network model [16]: Start with m_0 disconnected vertices, and grow the network by adding a vertex v with degree m per time step. An edge from the new vertex are added to an old vertex w with a probability proportional to w's degree.

We try to model the behavior of the cooperator density by starting from BA model networks with $m=m_0=3$ and 20 000 vertices. To tune how much mutually connected the vertices of highest degree are, we make the subset of the n_+ (or n_-) vertices of highest degree complete by adding the missing edges (or completely disconnected by removing existing edges). As seen in Fig. 6 the steps and spikes of the real-world networks can be qualitatively reproduced. The resemblance is closest for removed edges among a small number of hubs Fig. 6(c). If the edges are removed within a larger set of hubs, the steps are increasing in frequency and number until the structure is completely blurred out. If all missing edges are added within the top n_+ connected vertices, the steps vanishes but the spikes increase in size and width. An explanation is that when the vertices of highest degree are fully connected, a mutation of one of these would momentarily increase the number of cooperators by a large amount and also decrease the speed of restoration of cooperation. At the same time it is well known that cooperation is promoted in highly connected regions [17] which explains the high ρ value of the quasistable state.

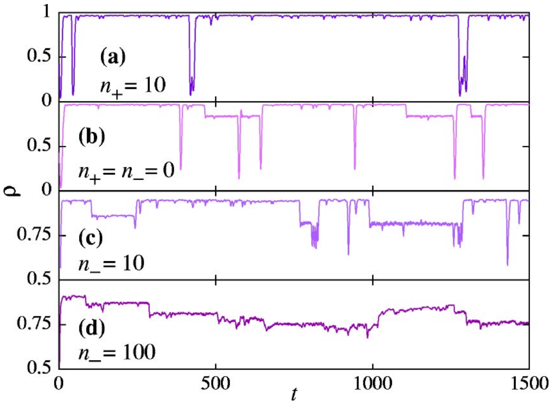

FIG. 6. Evolution of ρ for model networks. The network parameters are $m = m_0 = 3$, $N = 20\,000$. The dynamical parameters are $b = 1.4$ and $p_m = 0.001$.

In summary, we have studied Nowak and May's spatial PD game on empirical social networks. We find a complex time evolution of the cooperator level characterized by large spikes and transitions between a number of quasistable states. The quasistable states of low ρ are characterized by comparatively large gain for the defectors and large fraction of boundary edges, in quasistable states with high ρ this situation is reversed. We find that mutations on high-degree players are likely to be responsible for transitions between the quasistable states; but the outcome of a mutation of a high-degree vertex is also much dependent on the current configuration. These findings are presented for some specific sets of parameter values, but they are qualitatively the same for a broad range of values. We argue that the structure causing this behavior is a inhomogeneous degree distribution, and that the number of edges within the subset of highest-degree vertices is relatively low. Based on this observation we also construct model networks that reproduces the steps and spikes of the real-world networks. To epitomize, we believe that we have illustrated how underlying acquaintance patterns can give rise to a complex time development of social instability—a picture that could be empirically testable by carefully arranged social observations.

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